## A very simple property of Mersenne numbers

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- Version 0.1

Let show that: $2^{q}-1=1+6\left(1+2^{2}+2^{4}+2^{6}+\ldots+2^{2 \frac{q-1}{2}}\right)$.
Which is equivalent to :

$$
2^{q}-1=1+6 \sum_{i=0}^{\frac{q-1}{2}}\left(2^{i}\right)^{2}=1+2 \times 3 \sum_{i=0}^{\frac{q-1}{2}} 2^{2 i}
$$

First : $2^{q}-1=2^{q}-2+1=2\left(2^{q-1}-1\right)+1$.
Since $q$ is an odd number : $q-1=2 n$.
Now, show:

$$
2^{2 n}-1=3 \sum_{i=0}^{n-1} 2^{2 i}(I I)
$$

With $n=1$, we have: $2^{2 n}-1=3=3 \times 1=3\left(2^{2 \times 0}\right)$.
With $n=2$, we have: $2^{2 n}-1=15=3 \times 5=3\left(1+2^{2 \times 1}\right)=3\left(1+2^{2 \times(n-1)}\right)$. Thus, property $(I I)$ is true for ranks $n=1$ and 2 .

Suppose that property $(I I)$ is true at rank $n$.
Then, at rank $n+1$, we have: $2^{2(n+1)}-1=4 \times 2^{2 n}-1=2^{2 n}-1+3 \times 2^{2 n}$. And thus:
$2^{2(n+1)}-1=3 \sum_{i=0}^{n-1} 2^{2 i}+3 \times 2^{2 n}=3\left(2^{2 \times 0}+2^{2 \times 1}+\ldots+2^{2 \times(n-1)}+2^{2 \times n}=3 \sum_{i=0}^{n} 2^{2 i}\right.$
Which shows that the property $(I I)$ is also true at rank $n+1$ and thus that it is true for any $n>0$. And so the property $(I)$ is true for any odd $q>2$.

As an example: $2^{11}-1=1+2\left(\left(2^{5}\right)^{2}-1\right)=1+2 \times 3\left(1+2^{2}+4^{2}+8^{2}+16^{2}\right)$ which produces a nice figure: a square of side $1+2$ identical squares of side $2^{5}$ missing a square of side 1 each. And each of the nearly squares is made of 3 times a set of squares of side $\left(2^{i}\right)^{2}$ with $i=0 . .4$.

